Active microwave (radar) algorithm

7th IPWG Training Course

Nov. 17th, 2014

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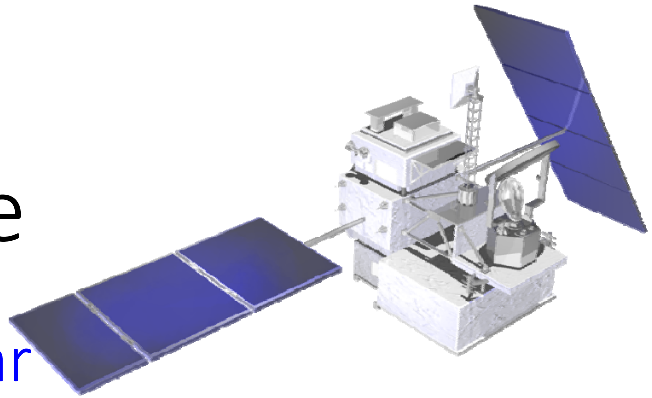
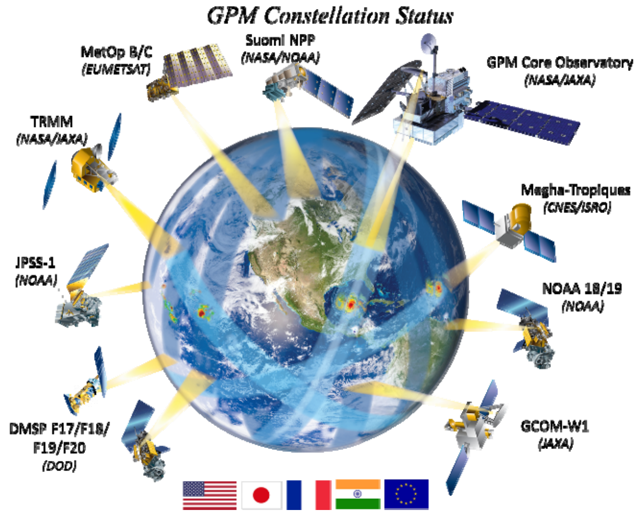
GPM Project





Core satellite

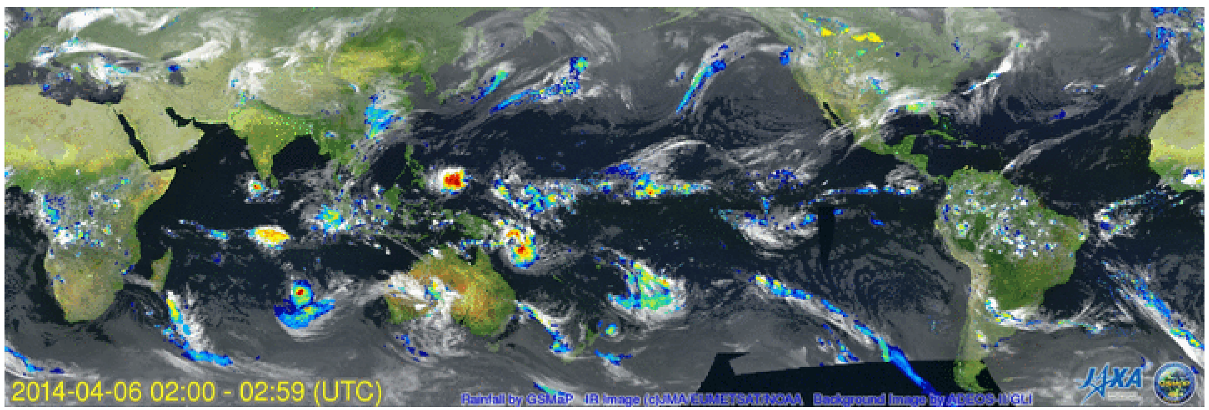
Dual-frequency Precipitation Radar (DPR)

GPM Microwave Imager (GMI)

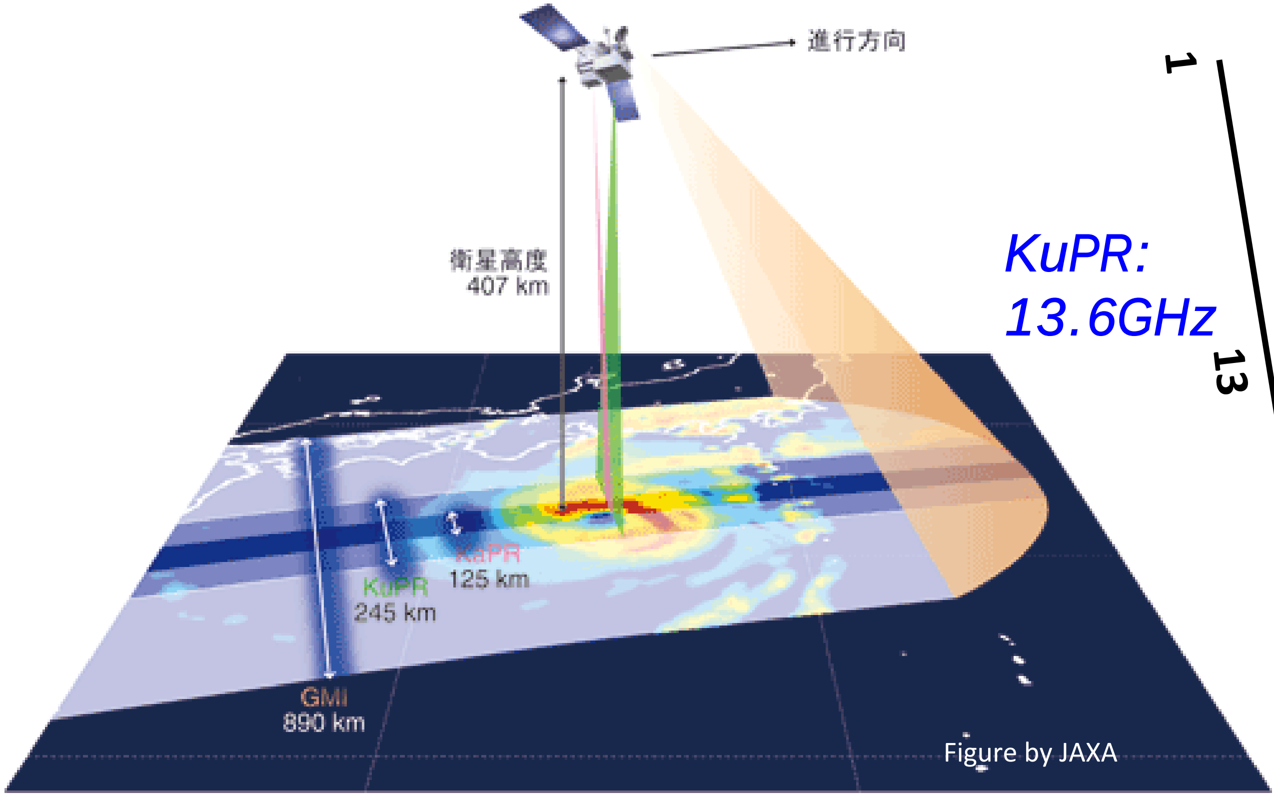
Constellation Satellites

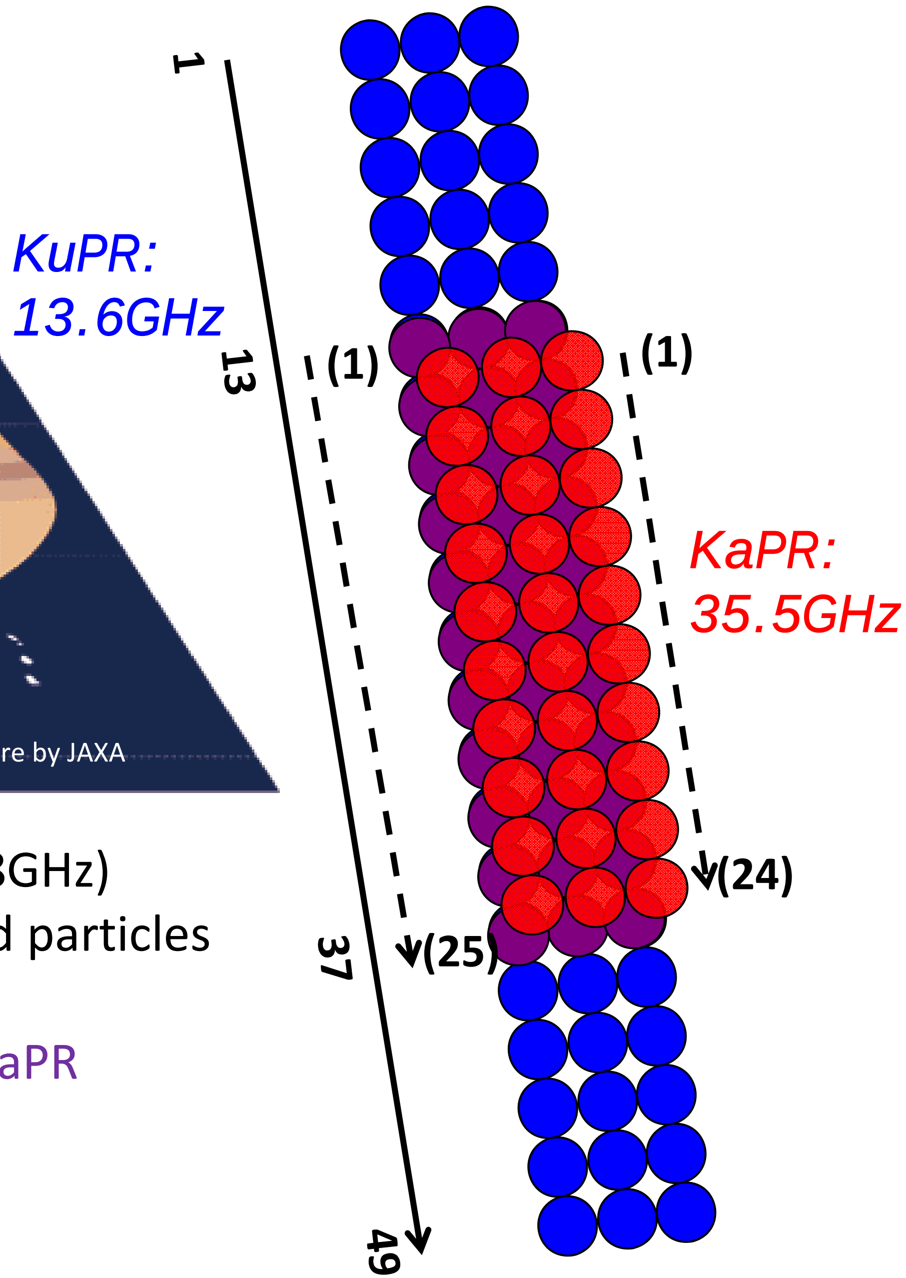
Microwave Imagers and Sounders

GSMaP



DPR (Dual-frequency Precipitation Radar)





KuPR(13.6GHz)	similar to TRMM/PR(13.8GHz)
KaPR(35.5GHz)	higher frequency for solid particles

Simultaneous measurement of KuPR and KaPR should give better precipitation estimates

Contents

- Introduction
- Quick review of drop size distribution

  drop size, $N(D)$, N_T, R, W, D_m, D_0, N_0, N_W

  exponential distribution, modified gamma distribution
- Variables related to radar measurement

  Z, Z_e, Rayleigh approximation, Mie theory

  Z-R relation, DFR method
- Basic idea of PR and DPR algorithms

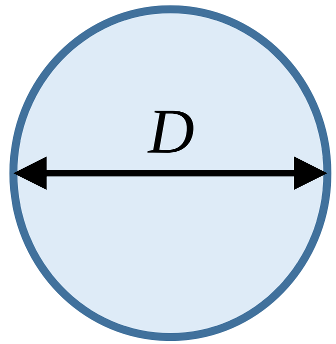
  attenuation correction, k, Z_f, k-Z_e relation

Quick Review of Drop Size Distribution

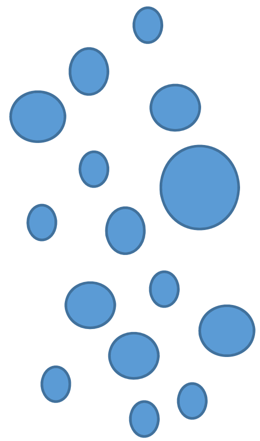
Section 1

Rain drops

Let us assume rain drops are sphere.
(Acutally, they are not perfectly sphere, but oblated.)
Drop size is the diameter of rain drop and is denoted by *D* [mm].
In most cases, *D* is between 0.1 mm to 7 mm.



DSD
(Drop Size Distribution)



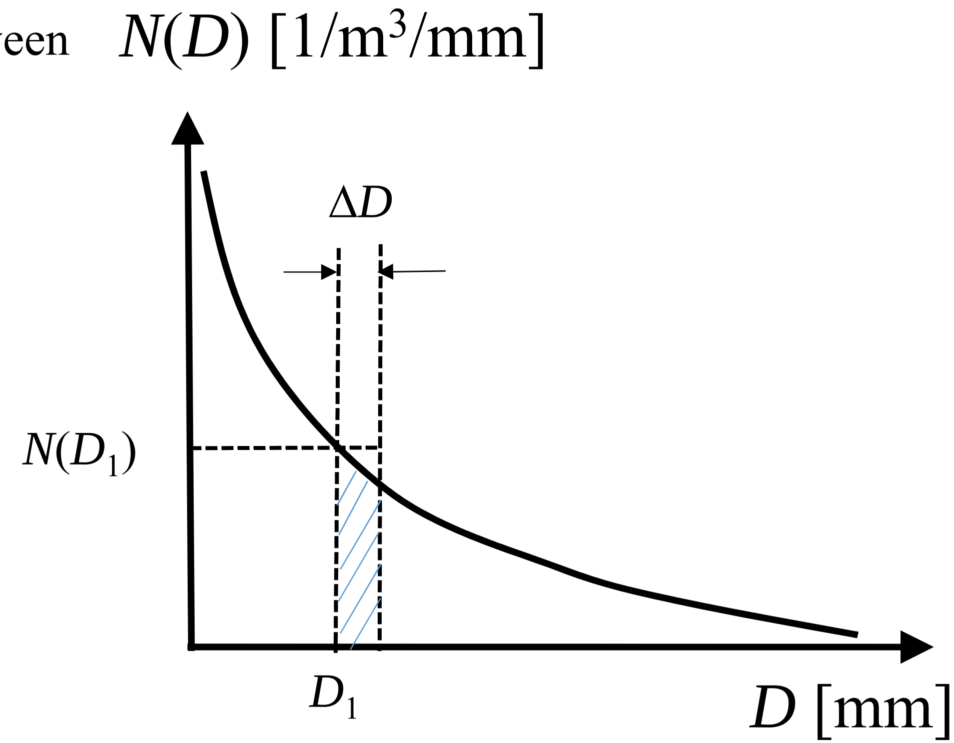
N is the number of rain drops in an unit volume.
N is a function of *D*, and the definition of *N* is given below.

The number of rain drops with *D* of between D_1 and $D_1+\Delta D$ in the volume of 1 m^3

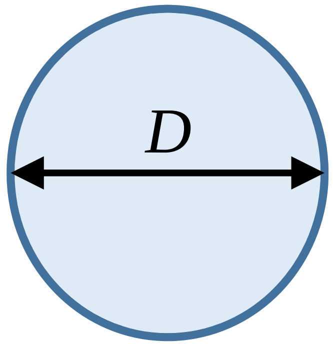
$$\int_{D_1}^{D_1+\Delta D} N(D)dD \approx N(D_1)\times\Delta D$$

The number of all rain drops in the volume of 1 m^3

$$N_T \equiv \int_{D_{\min}}^{D_{\max}} N(D)dD$$



DSD and rain rate



The total weight of rain drops [kg] in the volume of 1 m^3

$$W = \int_{D_{min}}^{D_{max}} \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \rho_w N(D)dD$$

The weight of rain drop [kg] with the size of *D*

ρ_w=density of water (=10^{-6} kg/mm^3)

The volume of rain drop [mm^3] with the size of *D*

Rain rate [mm/h]

$$R = 3.6\times10^{-3} \times \int_{D_{min}}^{D_{max}} \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 v(D)N(D)dD$$

$v(D)$=falling velocity [m/s]

The number of rain drops with the size of *D* and falling to the surface with area of 1 m^2 within 1 second.

Representative value of D

D_{m} (mass weighted drop diameter)

$$D_m = \frac{\int_{D_{\min}}^{D_{\max}} \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 D\rho_w N(D)dD}{\boxed{\int_{D_{\min}}^{D_{\max}} \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \rho_w N(D)dD}} = \frac{\int_{D_{\min}}^{D_{\max}} D^4 N(D)dD}{\int_{D_{\min}}^{D_{\max}} D^3 N(D)dD}$$

$=W$

D_0 (median volume diameter)

$$\int_{D_{\min}}^{D_0} \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \rho_w N(D)dD = \frac{1}{2}\boxed{\int_{D_{\min}}^{D_{\max}} \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \rho_w N(D)dD}$$

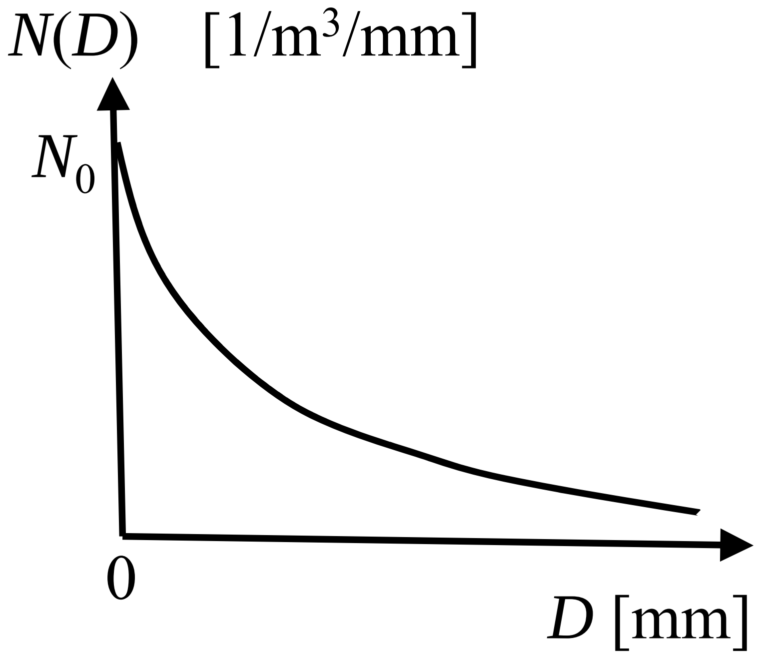
$=W$

$$\int_{D_{\min}}^{D_0} D^3 N(D)dD = \frac{1}{2}\int_{D_{\min}}^{D_{\max}} D^3 N(D)dD$$

DSD model (1)
Exponential Distribution

$$N(D) = N_0 \exp(-\Lambda D)$$

N_0 and Λ are DSD parameters



$$N_T = \int_0^\infty N_0 \exp(-\Lambda D) dD = \frac{N_0}{\Lambda}$$

$$W = \frac{\pi \rho_w}{6} \int_0^\infty N_0 D^3 \exp(-\Lambda D) dD = \frac{N_0 \pi \rho_w}{\Lambda^4}$$

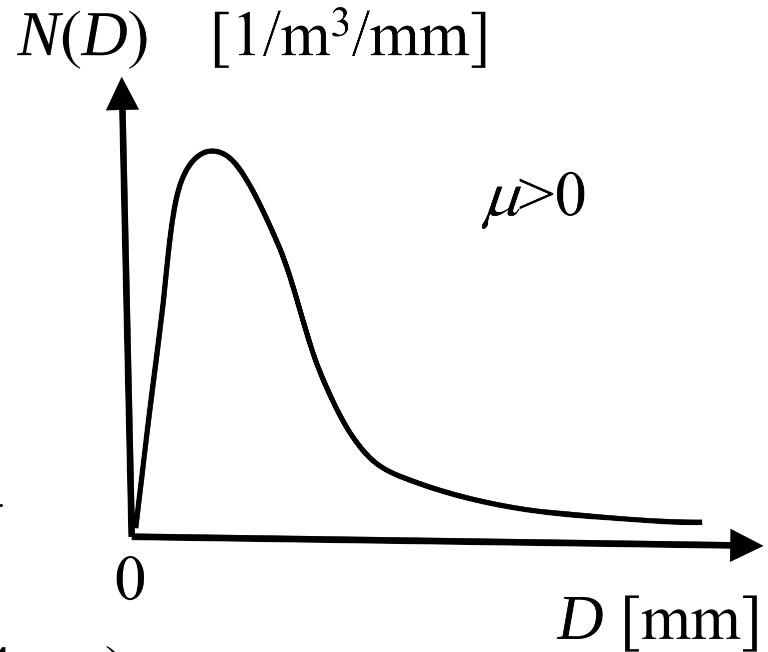
$$D_m = \frac{\int_0^\infty D^4 N_0 \exp(-\Lambda D) dD}{\int_0^\infty D^3 N_0 \exp(-\Lambda D) dD} = \left(\frac{4!}{\Lambda^5}\right) \Big/ \left(\frac{3!}{\Lambda^4}\right) = \frac{4}{\Lambda}$$

$$\int_0^\infty D^x \exp(-\Lambda D) dD = \frac{\Gamma(x+1)}{\Lambda^{x+1}}$$

$$\Gamma(x+1) = x\Gamma(x)$$

If x is an integer, $\Gamma(x+1) = x!$

DSD model (2)
Modified Gamma Distribution



$$N(D) = N_0 D^{\mu} \exp(-\Lambda D)$$

N_0, Λ, and μ are DSD parameters

$$N_T = \int_0^{\infty} N_0 D^{\mu} \exp(-\Lambda D) dD = \frac{N_0 \Gamma(\mu+1)}{\Lambda^{\mu+1}}$$

$$W = \frac{\pi \rho_w}{6} \int_0^{\infty} N_0 D^{3+\mu} \exp(-\Lambda D) dD = \frac{N_0 \pi \rho_w \Gamma(4+\mu)}{6 \Lambda^{4+\mu}}$$

$$D_m = \frac{\int_0^{\infty} D^{4+\mu} N_0 \exp(-\Lambda D) dD}{\int_0^{\infty} D^{3+\mu} N_0 \exp(-\Lambda D) dD} = \left(\frac{\Gamma(5+\mu)}{\Lambda^{5+\mu}} \right) \Bigg/ \left(\frac{\Gamma(4+\mu)}{\Lambda^{4+\mu}} \right) = \frac{4+\mu}{\Lambda}$$

In DPR algorithm, modified gamma distribution are adopted.

$$N(D) = N_0 D^{\mu} \exp(-\Lambda D)$$

$$f(\mu) = \frac{6(4+\mu)^{4+\mu}}{\Gamma(4+\mu) D_m{}^{\mu} 4^4}$$

$$N(D) = N_w f(\mu) D^{\mu} \exp\left(-\frac{(4+\mu)}{D_m} D \right)$$

Instead of N_0 and Λ, N_w and D_m are used as DSD parameters.

N_w is called "normalized intercept parameter".
W becomes independent of μ.

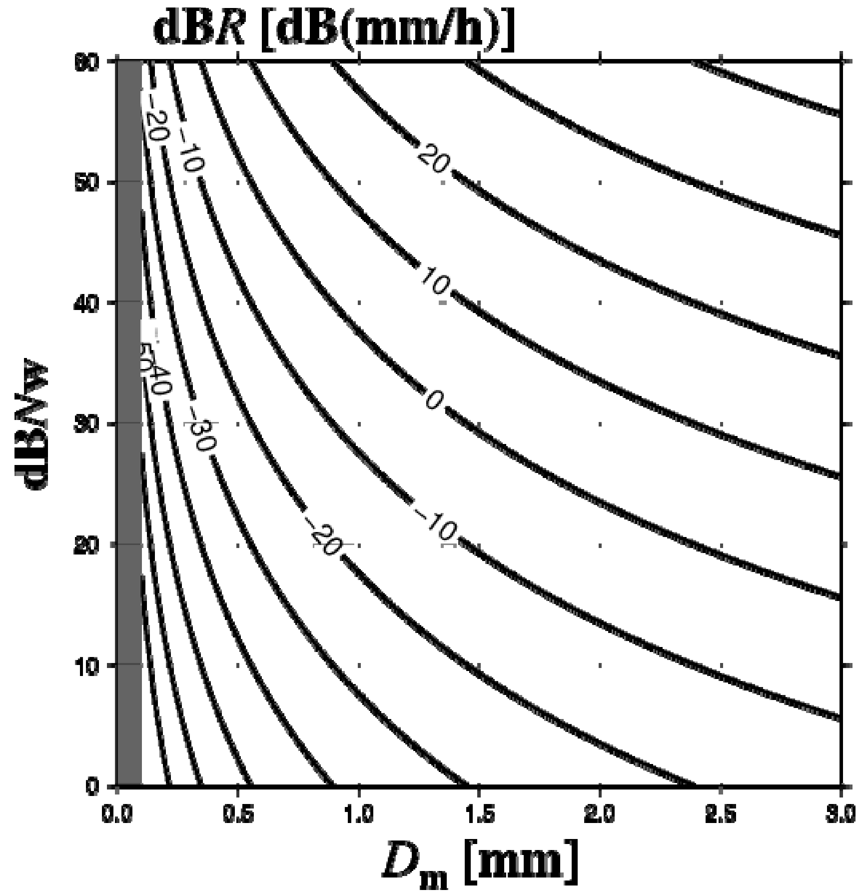
$$W = \frac{N_w \pi \rho_w D_m{}^4}{4^4}$$

DSD plane (1)

$$N(D) = N_w f(\mu) D^{\mu} \exp\left(-\frac{(4+\mu)}{D_m} D \right)$$

- In the current version of DPR algorithm, μ is fixed to be 3.
- DSD is represented by two parameters N_{w} and D_{m}.
- If N_{w} and D_{m} are determined, variables (such as W, R, N_{T}, D_0) can be calculated.
- In the right figure, D_{m} and $\mathrm{dB}N_{\mathrm{w}}=10\log_{10}(N_{\mathrm{w}})$ are abscissa and ordinate, respectively. Contours are $\mathrm{dB}R=10\log_{10}(R)$, where falling velocity is assumed as

$$v(D) = 4.854 \times D \times \exp(-0.195\, D)$$

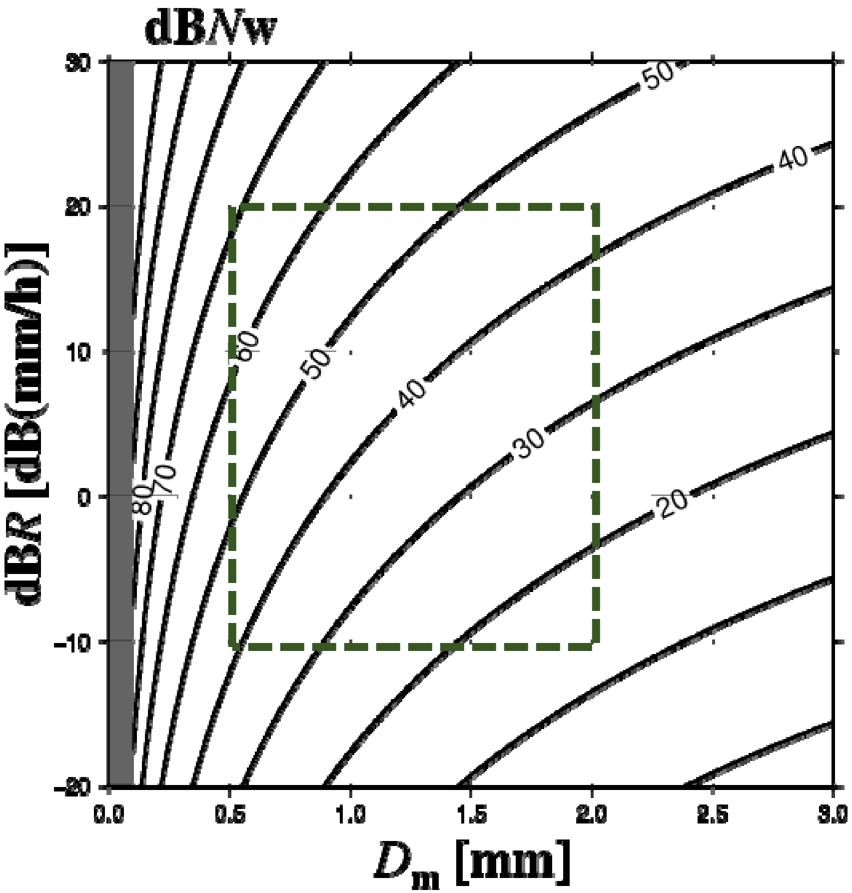


DSD plane (2)

$$N(D) = N_w f(\mu) D^{\mu} \exp\left(-\frac{(4+\mu)}{D_m} D \right)$$

- Instead of dBN_w, dBR is ordinate.
- Contours are dBN_w.
- This plane effectively shows feasible DSDs.
- D_m is usually 0.5 to 2.0 mm.
- R is usually 0.1 to 100 mm/h.

(-10dBmm/h to 20dBmm/h)



Variables related to Radar measurement

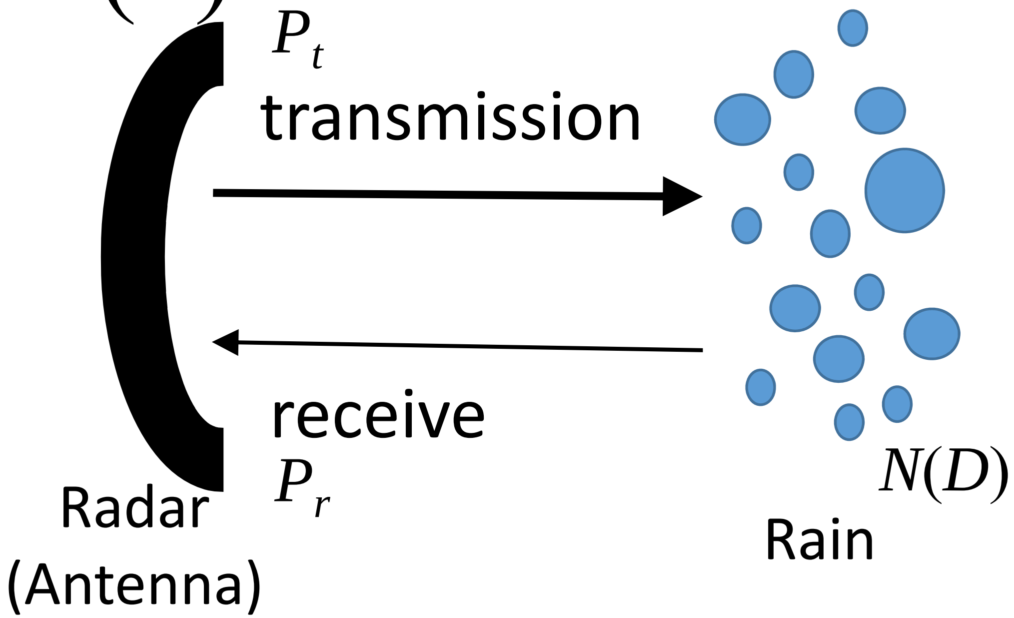
Section 2

Radar observation (1)

A radar transmits microwave.

Part of microwave radiation are backscattered by rain drops.

The radar receives returned microwave.



As long as Rayleigh approximation holds,

$$P_r = P_t C \int_{D_{\min}}^{D_{\max}} D^6 N(D) dD$$

C is a constant

Z (6^{th} moment of DSD) is called radar reflectivity factor

$$Z \equiv \int_{D_{\min}}^{D_{\max}} D^6 N(D) dD \qquad Z = \frac{P_r}{CP_t}$$

Z (radar reflectivity factor)

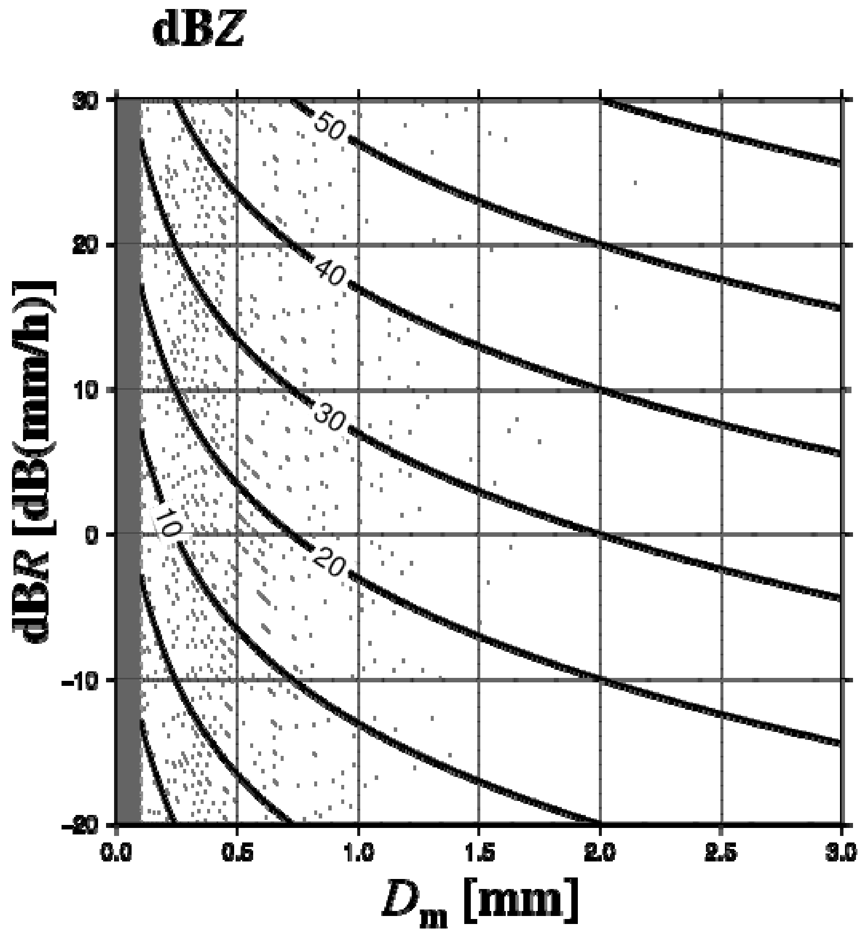
$$Z = \int_{D_{\min}}^{D_{\max}} D^6 N(D) dD$$

For modified gamma distribution,

$$Z = N_w f(\mu) \int_0^{\infty} D^{6+\mu} \exp\left(-\frac{(4+\mu)}{D_m} D\right) dD$$

$$Z = N_w f(\mu) \frac{\Gamma(7+\mu)}{(4+\mu)^{7+\mu}} D_m^{\ 7+\mu}$$

- In the right figure, $\mathrm{dB}Z = 10\log_{10}(Z)$ is shown by contours.
- Z is related not only to R but D_m.

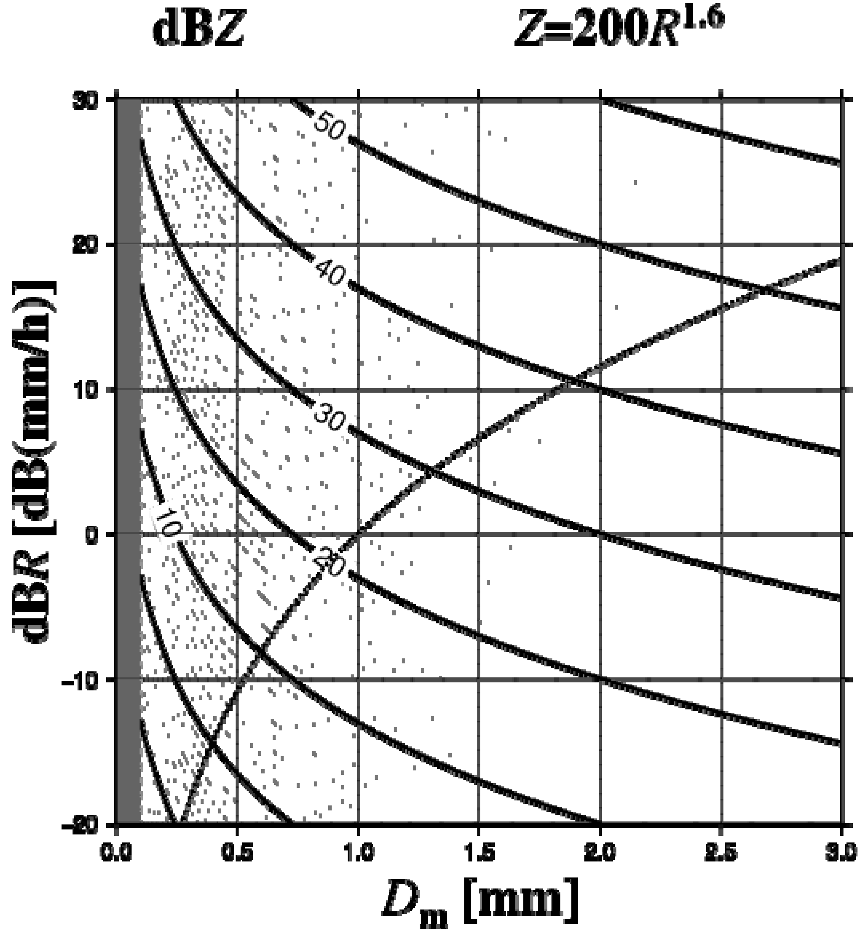


Z-*R* method

$$Z = aR^b$$

Z-*R* relation is assumed to follow a power law. For example, a=200 and b=1.6

- A fixed *Z*-*R* relation is valid only if (D_m, R) is on a line.
- For example, Z=200$R^{1.6}$ is valid if (D_m, R) is on the dotted line in the right figure.



Radar observation (2)

If the ratio of drop size to wavelength is large, Rayleigh approximation cannot be used.

$$P_r \neq P_t C Z$$

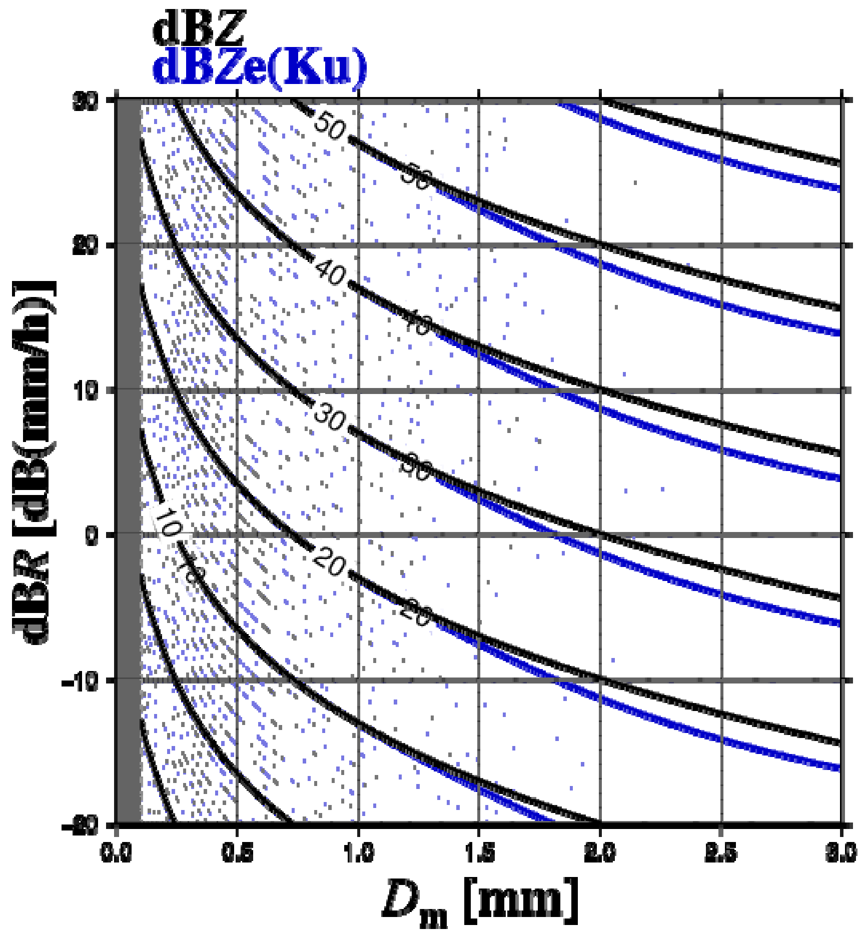
In this case, Z needs to be replaced by Z_e.
Z_e is called effective radar reflectivity factor.

$$P_r = P_t C Z_e$$

Z_e is calculated by Mie scattering theory. Z_e is dependent on drop size, wavelength of microwave, and refractivity index.

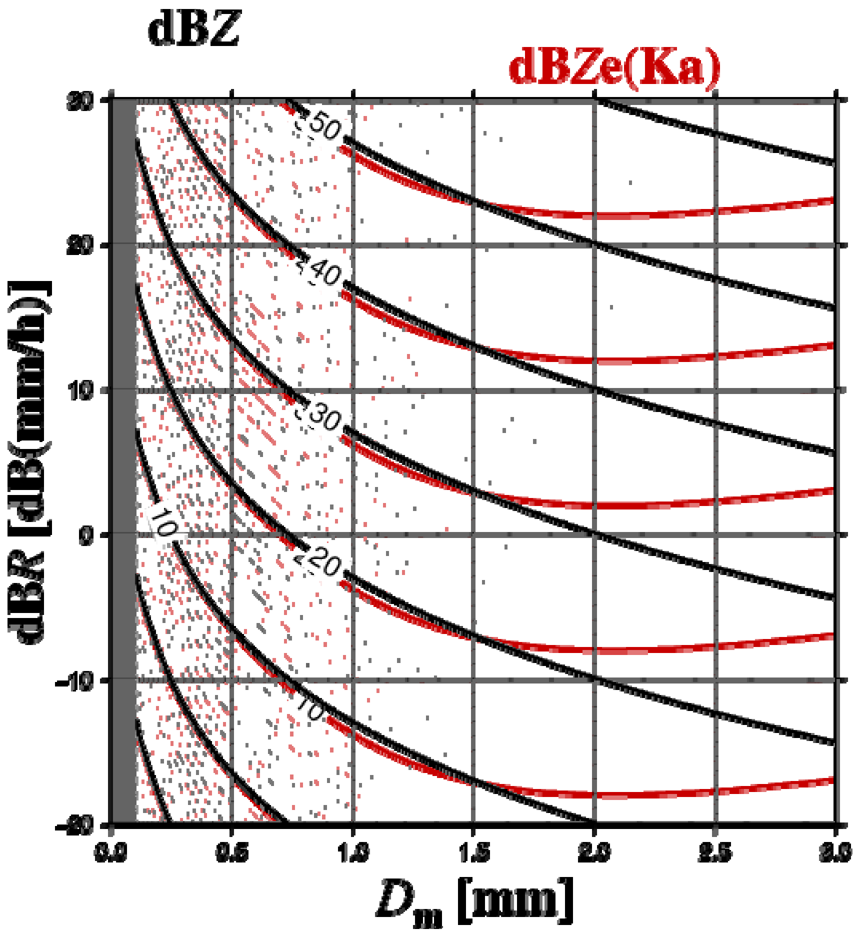
Z_e (KuPR)

- For the frequency of KuPR (13.6GHz)
- $dBZ_e = 10\log_{10}(Z_e)$ is shown by blue contours.
- The difference between Z and Z_e of 13.6GHz is clearly seen when D_m is larger than 1.5 mm.



Z_e (KaPR)

- For the frequency of KaPR (35.5GHz)
- $dBZ_e = 10\log_{10}(Z_e)$ is shown by red contours.
- The difference between Z and Z_e of 35.5GHz is seen when D_m is larger than 0.7 mm.

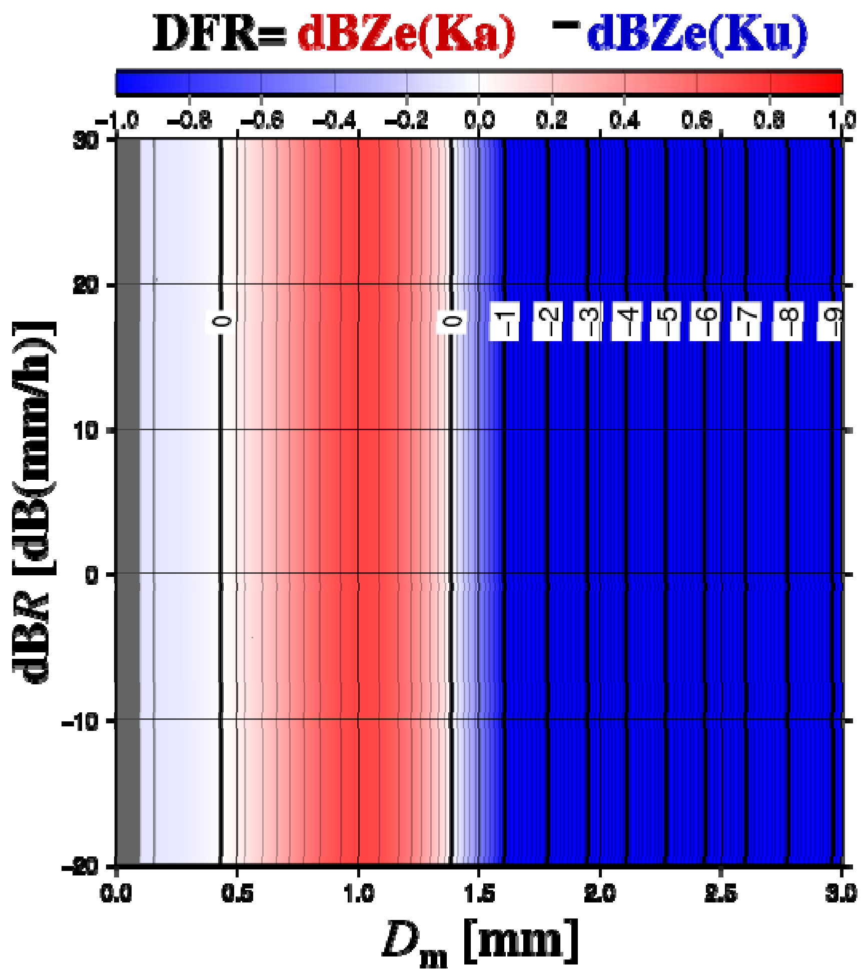


DFR method

- If we have Z_e at the two frequencies (KuPR and KaPR), we can retrieve R.
- In practice, the ratio of Z_e (KaPR) to Z_e (KuPR) or the difference between dBZ_e(KaPR) and dBZ_e(KuPR) is calculated.

DFR=dBZ_e(KaPR)-dBZ_e(KuPR)

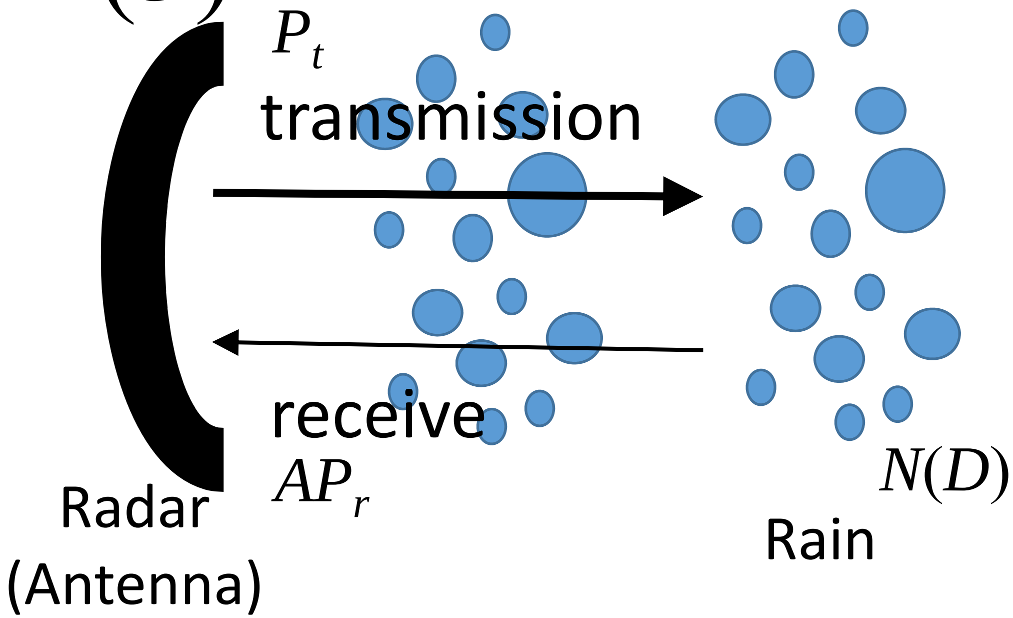
- DFR is dependent only on D_m.
- If D_m is determined by DFR, then R can be determined by D_m and Z_e.
- However, D_m is not uniquely determined if DFR is positive. In this case, larger D_m is usually selected.



Basic idea of PR and DPR algorithms

Section 3

Radar observation (3)



Ideally, Z_e can be given by radar measurement.

$$P_r = P_t C Z_e$$

Actually, measurement is affected by attenuation.

$$P_r = P_t C Z_e A \qquad (0<A<1)$$

So, Z_m or measured reflectivity factor is given instead of Z_e.

$$P_r = P_t C Z_m \qquad Z_m \equiv Z_e A$$

$$\mathrm{dBZ}_m = \mathrm{dBZ}_e - 10\log_{10} A^{-1}$$

Attenuation correction

$$\mathrm{dBZ}_m = \mathrm{dBZ}_e - 10\log_{10} A^{-1}$$

$$\mathrm{dBZ}_m = \mathrm{dBZ}_e - \sum_{i=1}^{N-1} 2k_i L - k_N L$$

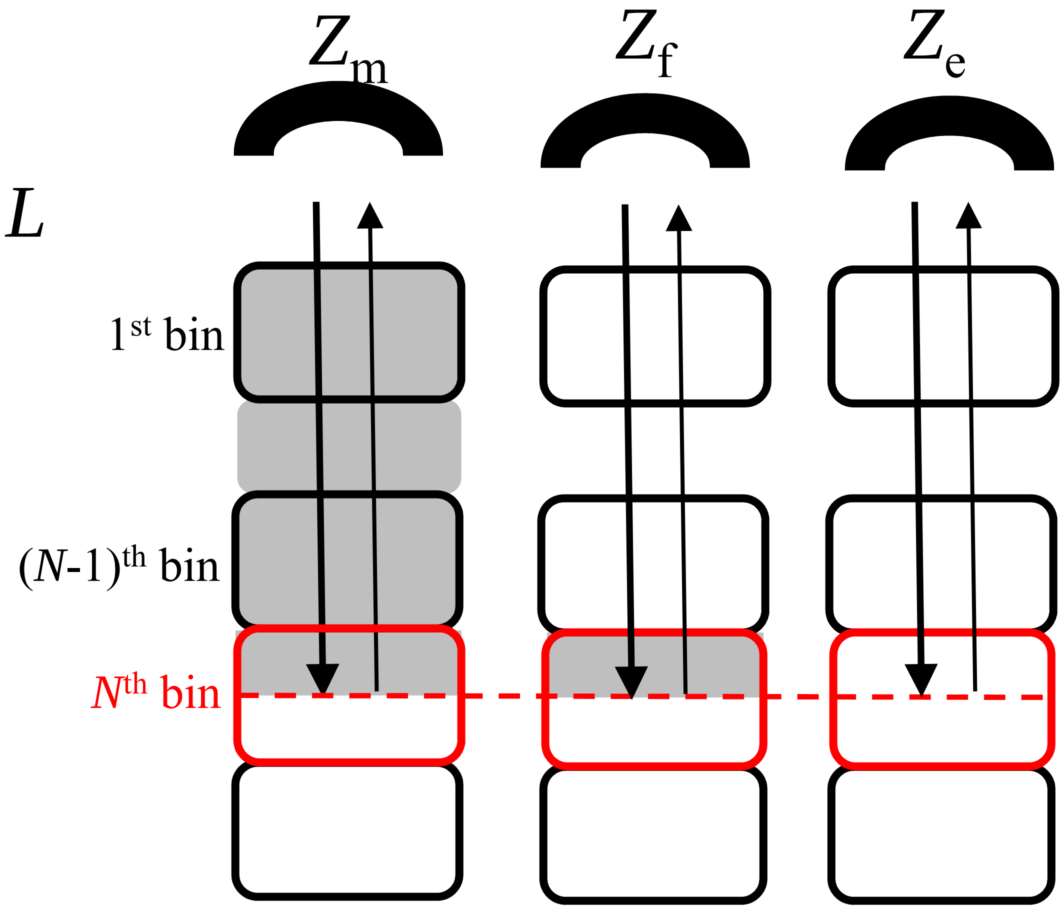
k specific attenuation at i^{th} bin
L thickness of each bin

If attenuation of 1st to (N-1)th bin is known, Z_f (defined as below) is calculated.

$$\mathrm{dBZ}_f \equiv \mathrm{dBZ}_m + \sum_{i=1}^{N-1} 2k_i L$$

Z_f is written as follows.

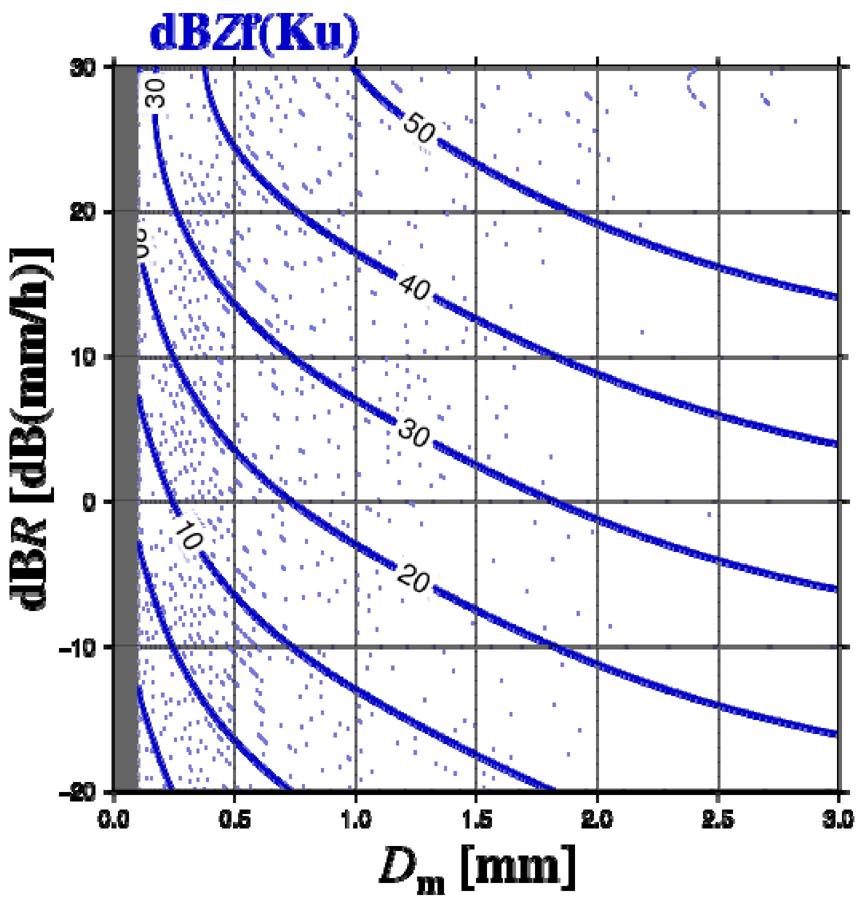
$$\mathrm{dBZ}_f = \mathrm{dBZ}_e - k_N L$$

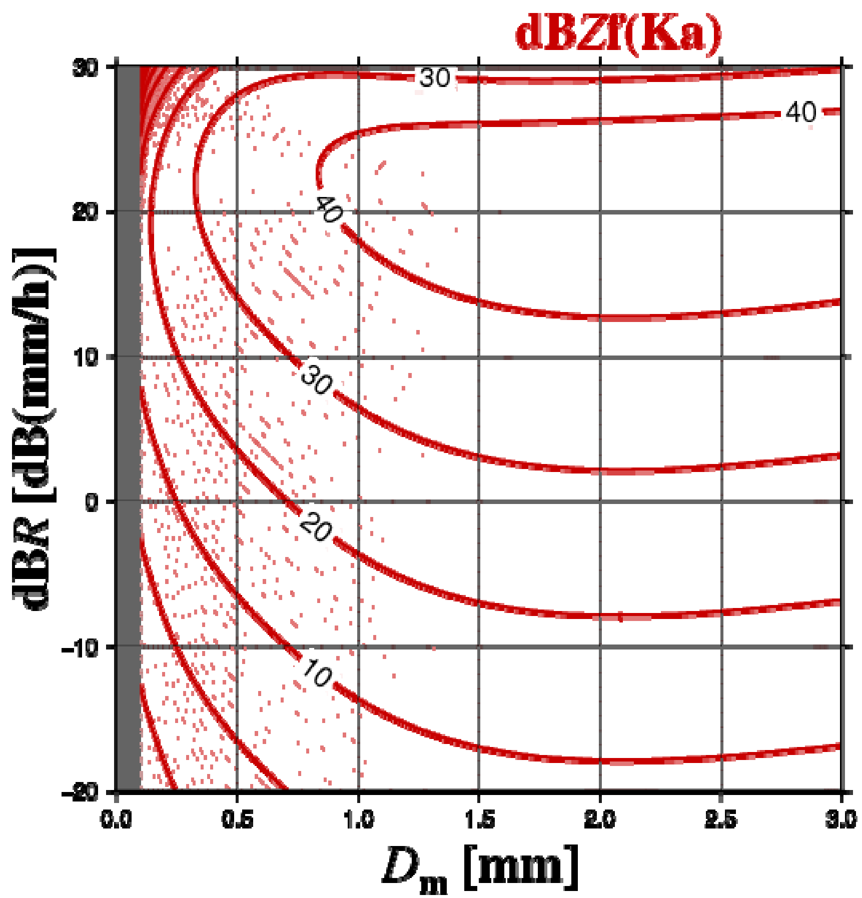


Z_f

$$\mathrm{dBZ}_f = \mathrm{dBZ}_e - k_N L$$

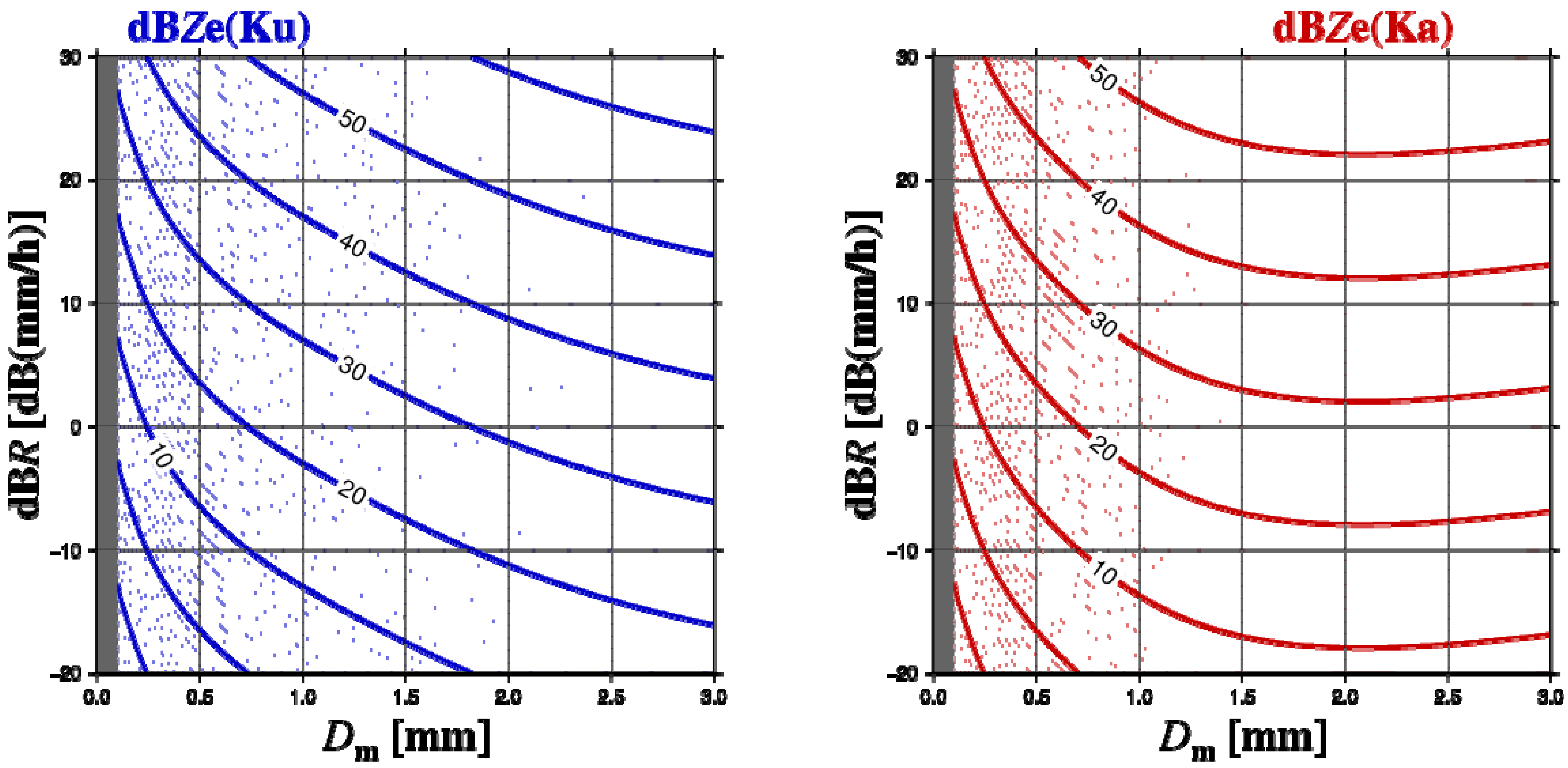
k_N is determined by DSD of the N^{th} bin. In case of spherical homogeneous particle, k_N can be calculated by Mie scattering theory as well as Z_e. So, dBZ_f is a function of D_m and R.





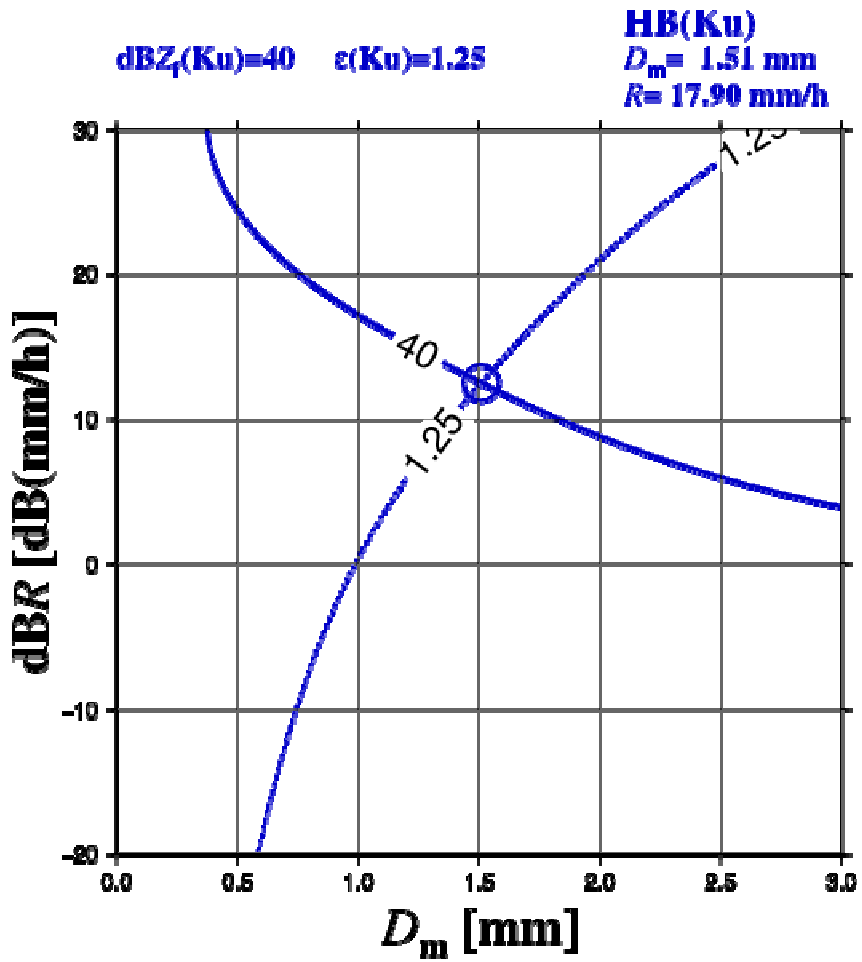
Z_e

Compare with Z_f



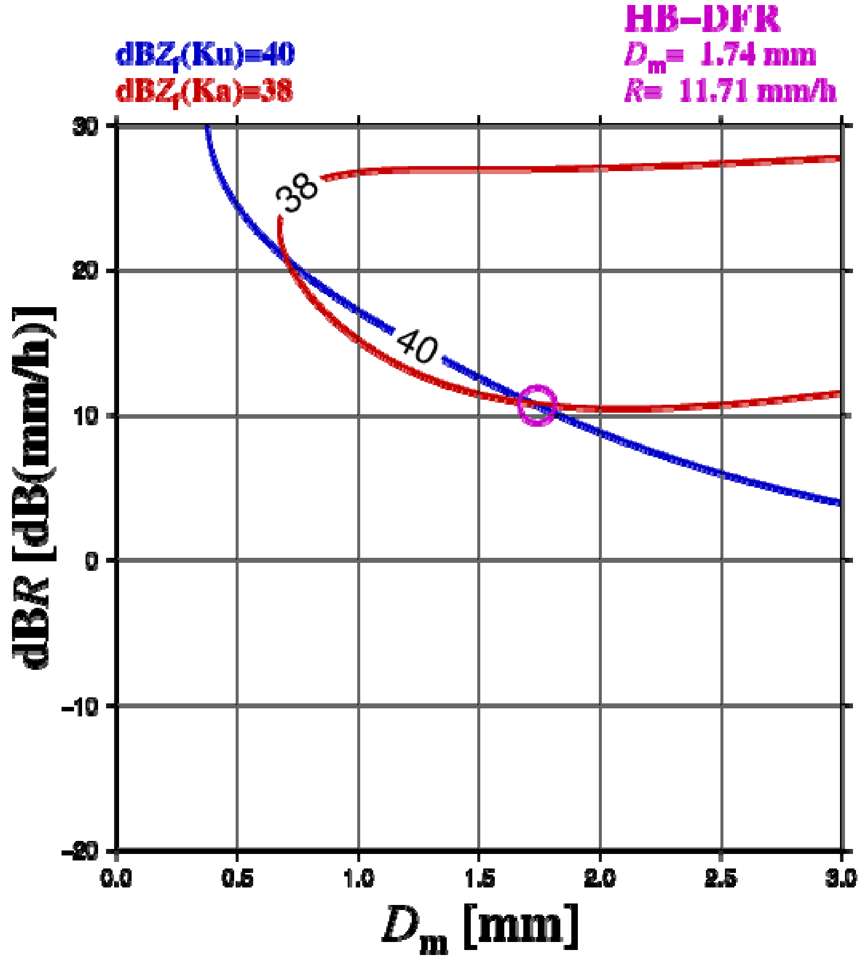
Basic idea of single-frequency algorithms (PR, KuPR, KaPR)

- Solve sequentially from upper bin to lower bin.
- At each bin, find the crossing point of the two lines.
  - Z_f (PR or KuPR or KaPR)
  - k-Z_e relation such as $k=\alpha Z_e^{\beta}$
- Similar with *Z-R* method.
- No universal k-Z_e relations. To reduce error in *R*, optimum k-Z_e relation should be selected.
  - α and β are dependent on rain type, particle phase and temperature.
  - α can be adjusted by surface reference technique.



Basic idea of dual-frequency algorithm

- Solve sequentially from upper bin to lower bin.
- At each bin, find the crossing point of the two lines.
  - Z_f(KuPR)
  - Z_f(KaPR)

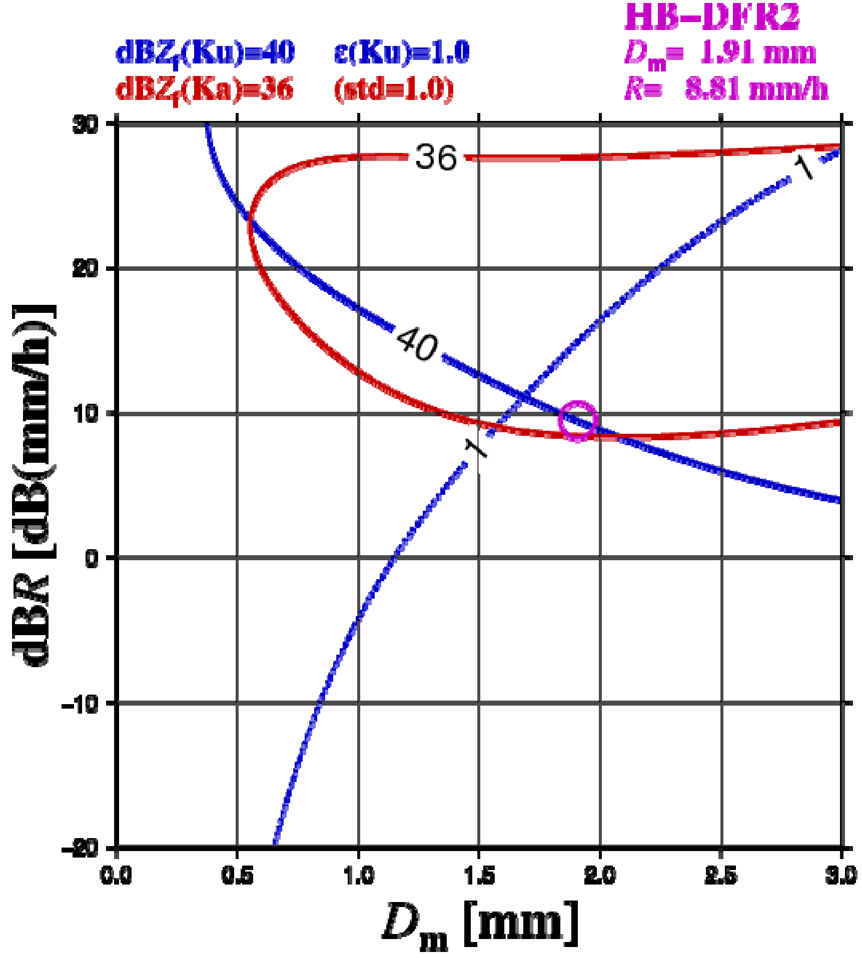


- Similar with DFR method.

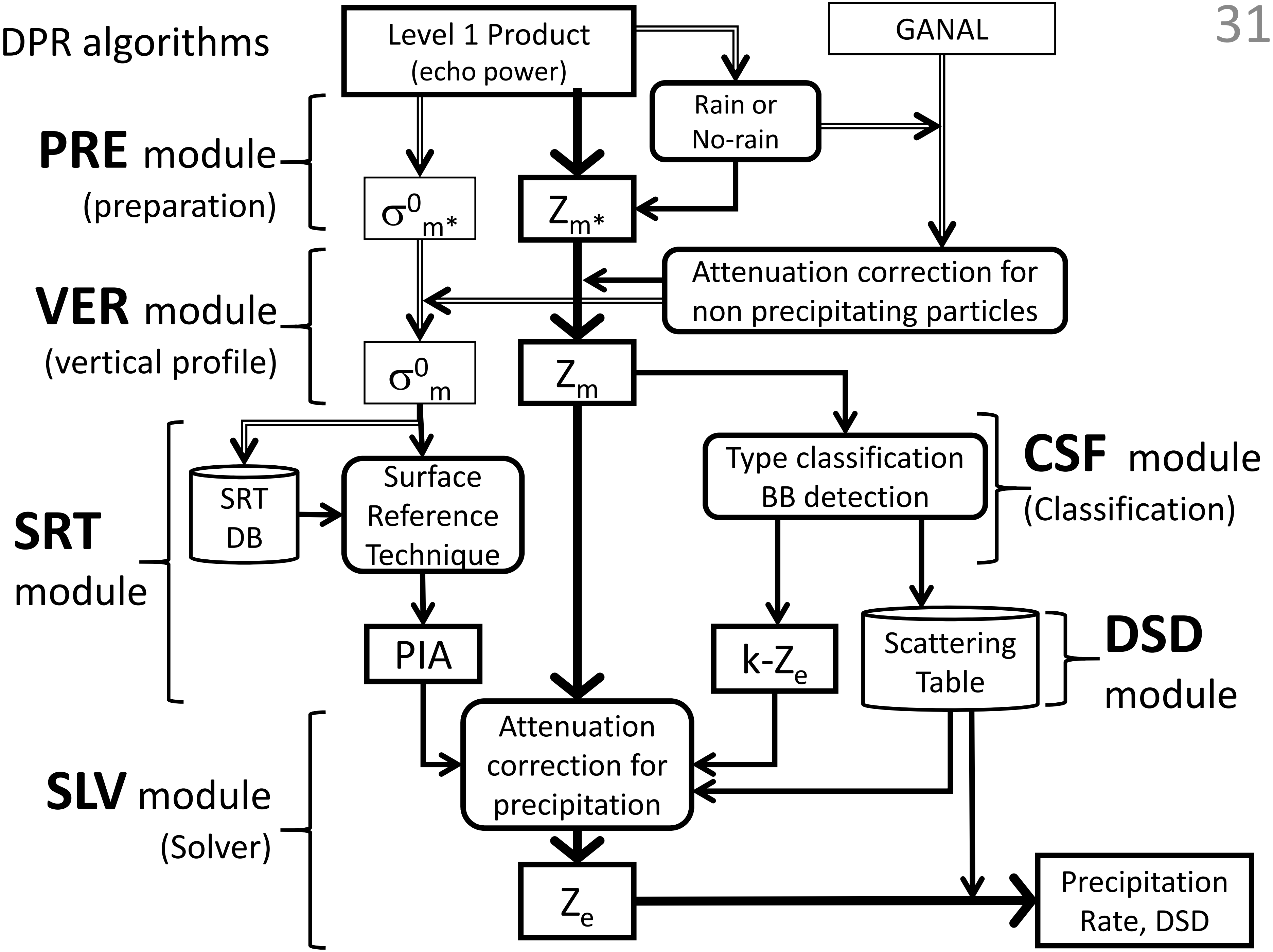
- This method has two weakness.
  - Always, two or more crossing points exist.→ Larger D_m (or smaller R) are selected.
  - Z_f (KaPR) has low reliability because attenuation is relatively heavy at KaPR. →Underestimation of R.

Improvement of dual-frequency algorithm

- Solve sequentially from upper bin to lower bin.
- At each bin, find the point closer to the three lines.
    - Z_f(KuPR)
    - Z_f(KaPR)
    - k-Z_e relation such as $k=\alpha Z_e^{\beta}$



DPR algorithms



More topics about PR and DPR

Black topics are explained today
Purple topics have been implemented in DPR algorithms
Blue topics were implemented in PR algorithms
Red topics have not been implemented.

- Retrieval of DSD and rain rates from Z_m
  - Drop shape: sphere oblate
  - DSD model: modified gamma distribution (μ=3)
  - Drop phase: liquid solid, mixed(melting)
- Attenuation correction for non-precipitating particles
- Bright band detection and rain type classification
- Clutter detection and extrapolation of estimates.
- Surface reference technique (SRT)
- Non uniform beam filling
- Multiple scattering